A Novel Globally Exponentially Stable Observer for Sensorless Control of the IPMSM via Kreisselmeier's Extension

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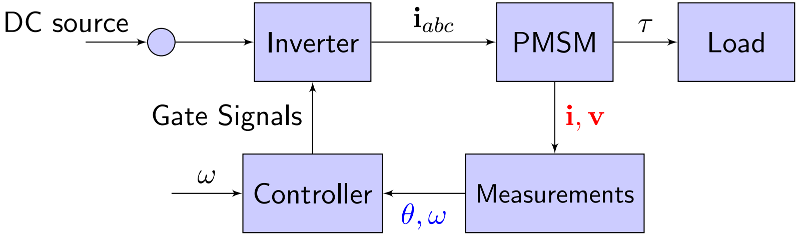
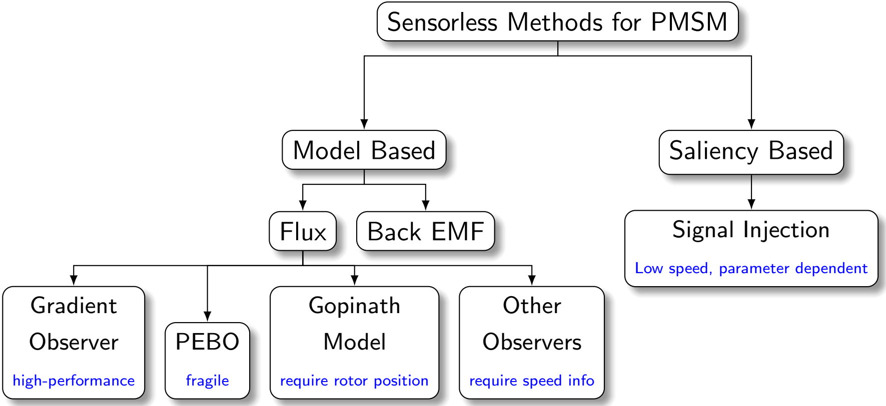
Background: Sensorless Control

- The control of PMSMs requires the mechanical variables, i.e. positions θ and velocities ω.
- Not available in many scenarios:
  - Low cost
  - Technological reasons (e.g. insufficient space in scooters)
- Solution: Sensorless control

$$[\text{ Outputs } \mathbf{i}, \mathbf{v} \ + \ \text{Model }] \quad \xrightarrow{\text{estimate}} \quad \theta, \omega$$



Sensorless Methods for PMSM



Two types of PMSMs:

1) Surface-mounted PMSM: Well-established for sensorless control
2) Interior PMSM:
    - Anisotropy of inductances
    - High power density, cheaper, reluctance torque
    - Few theoretical results reported

IPMSM Model

- Dynamics:

$$\dot{\lambda} = -R\mathbf{i} + \mathbf{v}. \tag{1}$$

- Output (current):

$$\mathbf{i} = \mathcal{L}^{-1}(\theta)[\lambda - \psi_m \mathbf{c}(\theta)] \tag{2}$$

- Functions:

$$\begin{aligned} \mathcal{L}(\theta) &:= L_s I_2 + \frac{L_0}{2} Q(\theta) \qquad [\ L_0 = 0 \implies \text{SPMSM}\ ] \\ Q(\theta) &:= \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix} \\ \mathbf{c}(\theta) &:= \mathsf{col}(\cos\theta, \sin\theta). \end{aligned}$$

- Active flux: $\mathbf{x} := \lambda - L_q \mathbf{i}$. (Bijection: $\lambda \leftrightarrow \mathbf{x}$, $\theta = \arctan \frac{\mathbf{x}_2}{\mathbf{x}_1}$)

Challenges: Highly nonlinear output function; Not in canonical forms.

Relevant Results

Related approaches with *provable* stability

1. (Ortega, Yi, Vukosavic, Nam and Choi, Automatica'21)
   First GES solution; Low gain for stability analysis, poor performance
2. (Malaizé and Praly, HAL'20)
   Simple design; GAS, complicated analysis
3. (Choi, Nam, Bobtsov and Ortega, TPEL'19)
   Good experimental results; Practical exponential stability
4. (Verrelli, Carfagna, Frigieri, Crinto and Lorenzani, Automatica'22)
   Parameter adaptation; Local stability

Strongest stability result: (Ortega et al., Automatica'21), relies on a **sufficiently small gain** for GES.

Question: A tunable GES flux observer?

Technical Lemma: Filtering

Lemma

The following (perturbed) linear regression equation

$$y = \Phi^\top \mathbf{x} + d + \epsilon_t \tag{3}$$

holds, with the *measurable* signals y and Φ given as

$$y := L_0 H_2[\mathbf{i}]^\top \Omega_1 + \frac{1}{\alpha}|\Omega_1|^2 + \frac{1}{\alpha} H_2[\Omega_2^\top \Omega_1], \quad \Phi := \Omega_1 + \Omega_2. \tag{4}$$

with $\Omega_1(t) := H_2[\mathbf{v} - R\mathbf{i} - L_q p\mathbf{i}]$, $\Omega_2(t) := H_2[\mathbf{v} - R\mathbf{i} - L_q p\mathbf{i}]$ and the filters

$$H_1(p) := \frac{\alpha p}{p+\alpha},\; H_2(p) := \frac{\alpha}{p+\alpha}, \tag{5}$$

where $\alpha > 0$ is a *tuning* parameter. The (unknown) perturbing signal d is

$$d := -\ell H_1\left[\mathbf{i}^\top \frac{\mathbf{x}}{|\mathbf{x}|}\right], \tag{6}$$

with $\ell := \psi_m L_0$, and $\epsilon_t \in \mathbb{R}^2$ is an exponentially decaying term.

Gradient Observer Structure

What do we have?

- The flux dynamics is measurable:

$$\dot{\lambda} = -R\mathbf{i} + \mathbf{v}. \tag{7}$$

- A perturbed linear regression model w.r.t. $\mathbf{x} = \lambda - L_q\mathbf{i}$:

$$y = \Phi^\top \mathbf{x} + d + \epsilon_t. \tag{8}$$

Gradient Flux Observer Structure

$$\begin{aligned} \dot{\hat{\lambda}} &= \mathbf{v} - R\mathbf{i} + E \\ \hat{\mathbf{x}} &= \hat{\lambda} - L_q\mathbf{i} \end{aligned} \tag{9}$$

The correction term is designed according to some optimal criteria:

(Choi et al., 19) Gradient of the cost function $J_1 = |y - \Phi^\top \hat{\mathbf{x}}|$

(Ortega et al., 21) Gradient with the compensation to d.

New design to overcome low gains?

New Regressor via Kreisselmeier's Extension

- The perturbed LRE: $y = \Phi^\top \mathbf{x} + d + \epsilon_t$.
- Applying an LTV filter to the both sides (Kreisselmeier, TAC'77):

$$
\begin{aligned}
\dot{Q} &= -a(Q - \Phi\Phi^\top),\ Q(0) = 0 \\
\dot{Y} &= -a(Y - \Phi e) + \boxed{QE},\ Y(0) = 0 \\
E &= -\gamma Y,
\end{aligned}
\tag{10}
$$

  with gains $\gamma, a > 0$.

- The term QE compensates the dynamics of $\mathbf{x}$; for parameter estimation (i.e. $\mathbf{x} = \texttt{const}$), we do *not* need QE for KRE.

The existence of an invariant manifold

$$\mathcal{M} := \{(Y, Q, \xi) \in \mathbb{R}^2 \times \mathbb{R}^{2\times 2} \times \mathbb{R}^2 :\ Y = Q\tilde{\mathbf{x}} + \xi\} \tag{11}$$

with

$$\dot{\xi} = -a(\xi - \Phi(\hat{d} - d)), \quad \xi(0) = 0. \tag{12}$$

Active Flux/Position Observer Design

$$
\left.\begin{aligned}
\dot{Q} &= -a\big(Q - \Phi\Phi^\top\big),\ Q(0) = 0 \\
\dot{Y} &= -a\big(Y - \Phi e\big) + QE,\ Y(0) = 0 \\
E &= -\gamma Y
\end{aligned}\right\} \text{(KRE)}
$$

$$
\left.\begin{aligned}
\dot{\hat{\lambda}} &= \mathbf{v} - R\mathbf{i} + E \\
\hat{\mathbf{x}} &= \hat{\lambda} - L_q\mathbf{i} \\
\hat{\theta} &= \mathrm{atan2}(\hat{\mathbf{x}}_2, \hat{\mathbf{x}}_1),
\end{aligned}\right\} \text{(Flux-position estimate)} \tag{13}
$$

with the estimate of the disturbance signal

$$
\hat{d} \;=\; -\ell H_1[\mathbf{i}^\top \sigma(\hat{\mathbf{x}})], \tag{14}
$$

the variable

$$
e := \Phi^\top \hat{\mathbf{x}} + \hat{d} - y,
$$

and the mapping

$$
\sigma(\hat{\mathbf{x}}) = \begin{cases} \dfrac{\hat{\mathbf{x}}}{|\hat{\mathbf{x}}|} & \text{if } \ |\hat{\mathbf{x}}| \geq \epsilon > 0 \\ \text{col}(0,0) & \text{otherwise,} \end{cases}
$$

where $\epsilon \in (0, x_{\mathtt{min}})$, and $a, \alpha, \gamma > 0$ are tuning parameters. □

Standing Assumptions

Assumption (Motor rotating behavior)

Φ is persistently excited, i.e. $\exists T > 0, \delta > 0$ s.t.

$$\int_t^{t+T} \Phi(s)\Phi^\top(s)ds \geq \delta I_2, \quad \forall t \geq 0 \tag{15}$$

with $|\Phi| < +\infty$.

Assumption (Boundedness)

The motor operates in a mode s.t.

- $\mathbf{i}, \mathbf{v}, \lambda \in L_\infty$;
- $|\mathbf{x}| \geq x_{\mathtt{min}}$ for some $x_{\mathtt{min}} > 0$.

Assumption (Small anisotropy)

The current $\mathbf{i}$ verifies $|L_0 \mathbf{i}| < \psi_m$.

Error Dynamics

- Due to the invariant manifold $\mathcal{M} = \{(Y, Q, \xi) : \; Y = Q\tilde{\mathbf{x}} + \xi\}$, the overall error dynamics can be represented by the state

$$\chi := \mathrm{col}(\tilde{\mathbf{x}}, \xi, z) \in \mathbb{R}^5 \tag{16}$$

with z the state of the filter $H_1[\cdot] = \frac{\alpha p}{p+\alpha}[\cdot]$.

- The overall dynamics:

$$\dot{\chi} = A(t)\chi + \alpha\Delta(t)\chi,$$

where we defined the matrices

$$\begin{aligned} A(t) &:= \begin{bmatrix} -\gamma Q(t) & -\gamma I_2 & 0_{2\times 1} \\ 0_{2\times 2} & -aI_2 & a\Phi(t) \\ 0_{1\times 2} & 0_{1\times 2} & -\alpha \end{bmatrix} \in \mathbb{R}^{5\times 5} \\ \Delta(t) &:= \begin{bmatrix} 0_{2\times 2} & 0_{2\times 2} & 0_{2\times 1} \\ -a\Phi(t)\mathbf{w}^\top(t) & 0_{2\times 2} & 0_{2\times 1} \\ \alpha\mathbf{w}^\top(t) & 0_{1\times 2} & 0 \end{bmatrix} \in \mathbb{R}^{5\times 5} \end{aligned}$$

- KRE makes the $(1,1)$-element negative definite $\forall t$. We are able to tune convergence speed!

Main Results

Proposition

Consider the IPMSM model with output $\mathbf{i}$. For any adaptation gains $\gamma > 0$ and $a > 0$, $\exists \alpha_{\max} > 0$ s.t. the proposed observer guarantees the global exponential convergence $\hat{\mathbf{x}} \to \mathbf{x}$ as $t \to \infty$ and

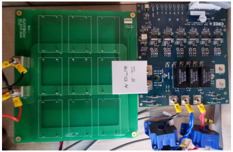
$$\lim_{t \to \infty} \left| \hat{\theta}(t) - \theta(t) \right| = 0 \quad \text{(exp.)}$$

for any $\alpha \leq \alpha_{\max}$.

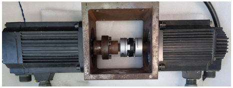
- Strongest stability result on sensorless observers for IPMSMs;
- Also works perfectly in practice!
- The correction term E can be roughly viewed as the gradient of the cost function from KREs.

Experimental Setup

- Rotated at 1000 rev/min
- Inverter was made with Silicon Carbide Six-Pack Power Modules (CCS050M12CM2) and gate driver (CGD15FB45P1)
- Controlled by DSP TMS320F28337
- Switching frequency: 5 kHz
- Sampling frequency: 10 kHz



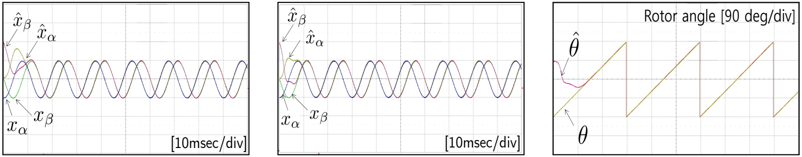
(a) SiC MOSFET Based Inverter



(b) Dynamo set

Figure 1: Test platform

Experimental Results

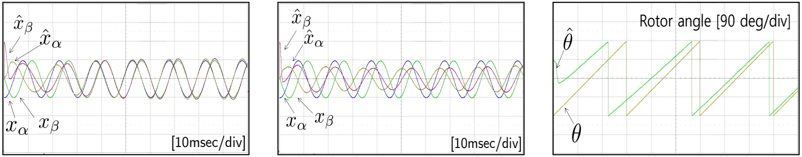


(a) Estimation of rotor flux with $\gamma = 1$

(b) Estimation of rotor flux with $\gamma = 5$

(c) Estimation of rotor angle with $\gamma = 5$

Figure 2: Experimental performance of the proposed observer



(a) Estimation of rotor flux with $\gamma = 1$

(b) Estimation of rotor flux with $\gamma = 5$

(c) Estimation of rotor angle with $\gamma = 5$

Figure 3: Performance of the observer in (Ortega et al., Auto'21)

Thank you!